

The concept of vacuum in quantum electrodynamics

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Abstract

A re-evaluation of the concept of vacuum in quantum electrodynamics is presented, focusing on the vacuum of the quantized electromagnetic field. In contrast to the ‘nothingness’ associated to the idea of classical vacuum, subtle aspects are found in relation to the vacuum of the quantized electromagnetic field both at theoretical and experimental levels. These are not the usually called vacuum effects. The view defended here is that the so-called vacuum effects are not due to the ground state of the quantized electromagnetic field.

1 Introduction

There is a widespread idea that we can associate with the concept of quantum vacuum important measurable consequences, even, like in the case of the commonly accepted interpretation of the Casimir effect, clear dynamical effects. To better address the problematic of what vacuum concept we really have in quantum electrodynamics, I start in section 2 with a brief presentation of the quantized electromagnetic field. The quantized electromagnetic field’s vacuum or ground state is the state with the lowest energy, corresponding to no (transverse) photons present.

Due to the quantization, in all the quantum electromagnetic field states corresponding to a defined number of quanta, the variance of the electric and magnetic fields is not zero.¹ This situation also occurs in the ground state (or vacuum state). This has measurable consequences. This is in clear contrast to the classical counterpart where the vacuum state corresponds to a null electromagnetic field in some region of space. Also, there are more formal aspects differentiating the classical and quantum vacuum. In section 3, to clarify things, first I address the quantum electrodynamical description of the interaction of radiation and matter. In particular I consider the problem of the divergence in the perturbative calculations within quantum electrodynamics as first noticed by Freeman Dyson in 1951. We will see that this result has implications in relation to the definition of the concept of vacuum in quantum electrodynamics.

In this section, I will focus on the properties of the ground state of the quantized electromagnetic field. Following Peter Milloni I will make a case for an interpretation of the Casimir effect that does not rely on zero-point energy fluctuations. This does not mean that the vacuum of the quantized electromagnetic field can be disposed of. Contrary to the classical case where it is possible to consider charged matter in an empty region of space – where there is no external electromagnetic field but only the

¹ In section 3.3 I will make the case for an interpretation of the variance in terms of a statistical spread (distribution) in the results of independent measurements made on identically prepared systems. This interpretation is made within the broader framework of the ‘ensemble’ interpretation of quantum mechanics.

field of the charged matter itself –, in the case of quantized fields this is no longer possible. For the formal consistency of the theory we must consider that a charged particle is always interacting with an *external* quantized electromagnetic field even if just in its ground state when considering the charged particle in a empty region of space. In this way, together with charged matter, we must always consider at least an ‘empty-space’ or ‘space-vacuum’ field in its vacuum or ground state with its associated non-zero variance, which can be related to experimental results. In fact using the balanced homodyne detection method it is possible to obtain experimental results corresponding to the non-vanishing variance of the vacuum state.

It seems then that we cannot recover the ‘nothingness’ of the classical notion, but, nevertheless, the physical properties we can really associate with the vacuum concept are much more subtle than usually thought, and do not present any experimental particularities that are not found in all quantum states corresponding to a defined number of quanta: the non-vanishing variance is a common characteristic of all these states, not only the ground state. In this way, in this paper, I will be making a case for a statistically empirically demonstrable notion of the vacuum in quantum electrodynamics independent of dynamical fluctuations.

2 The quantization of the electromagnetic field

In simple terms, the quantization of the electromagnetic field can be seen as follows. In the case of an electromagnetic field in a region free of charges, The Maxwell-Lorentz equations are:

$$\nabla \cdot \mathbf{E} = 0 \quad (2.1)$$

$$\nabla \times \mathbf{B} = \frac{\partial \mathbf{E}}{c \partial t} \quad (2.2)$$

$$\nabla \cdot \mathbf{B} = 0 \quad (2.3)$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{c \partial t} \quad (2.4)$$

From these equations we see that the electric and magnetic fields (strengths) can be defined in terms of a scalar and vector potentials $\phi(\mathbf{x}, t)$ and $\mathbf{A}(\mathbf{x}, t)$:

$$\mathbf{B} = \nabla \times \mathbf{A} \quad (2.5)$$

$$\mathbf{E} = -\nabla \phi - \frac{\partial \mathbf{A}}{c \partial t} \quad (2.6)$$

This enables to write down Maxwell-Lorentz equations for free space in terms of the scalar and vector potentials. In a four-vector notation we have that the Maxwell-Lorentz equations become

$$\left(\frac{\partial^2}{c \partial t^2} - \nabla^2 \right) A_\mu - \frac{\partial}{\partial x_\mu} \left(\frac{\partial A_\nu}{\partial x_\nu} \right) = 0 \quad (2.7)$$

The potentials are not uniquely determined, since it is possible to leave the fields unaltered when doing the following transformation

$$A_\mu \rightarrow A'_\mu = A_\mu + \frac{\partial \Lambda(x)}{\partial x_\mu} \quad (2.8)$$

(where $\Lambda(x)$ is an arbitrary function). This means the theory is invariant regarding what is called a gauge transformation (of the second kind). The Maxwell-Lorentz equations in free space can be simplified in a manifestly covariant way by taking the potential to satisfy the so-called Lorentz condition $\partial_\mu A^\mu(x) = 0$. In this case the Maxwell-Lorentz equations reduce to the wave equation $\square A_\mu = 0$ (the d'Alembert equation in the free charge case).

In the quantization of the electromagnetic field we take initially the components of the four-vector-potential to be independent, that is, we disregard the Lorentz condition. We then make a Fourier expansion (in terms of a complete set of solutions of the wave equation) of the free electromagnetic field (as given by the four-vector potential) $A^\mu(x)$:

$$A^\mu(x) = A^{\mu+}(x) + A^{\mu-}(x) \quad (2.9)$$

$$A^{\mu+}(x) = \sum_{\mathbf{k}} \sum_{r=0}^3 \left(\frac{\hbar c^2}{2V\omega_{\mathbf{k}}} \right)^{1/2} \epsilon_r^\mu(\mathbf{k}) a_r(\mathbf{k}) e^{-ikx} \quad (2.10)$$

$$A^{\mu-}(x) = \sum_{\mathbf{k}} \sum_{r=0}^3 \left(\frac{\hbar c^2}{2V\omega_{\mathbf{k}}} \right)^{1/2} \epsilon_r^\mu(\mathbf{k}) a_r^*(\mathbf{k}) e^{ikx} \quad (2.11)$$

For each \mathbf{k} there are four independent mutually orthogonal unit vector or 'polarizations'. It is useful to choose the vectors as given by $\epsilon_0^\mu(\mathbf{k}) = (1, 0, 0, 0)$, and $\epsilon_r^\mu(\mathbf{k}) = (0, \epsilon_r(\mathbf{k}))$, with $r = 1, 2, 3$, where $\epsilon_1(\mathbf{k})$ and $\epsilon_2(\mathbf{k})$ are mutually orthogonal unit vector also orthogonal to \mathbf{k} , and $\epsilon_3(\mathbf{k})$ is a unit vector longitudinal to \mathbf{k} . With this choice, the vector-potential dependent on $\epsilon_1^\mu(\mathbf{k})$ and $\epsilon_2^\mu(\mathbf{k})$ refers to transversely polarized light; the vector-potential dependent on $\epsilon_3^\mu(\mathbf{k})$ refers to a longitudinal polarization; and the vector-potential dependent on $\epsilon_0^\mu(\mathbf{k})$ refers to a so-called scalar or 'time-like' polarization (Bogoliubov and Shirkov 1959, 55-57; Källén 1972, 19).

Up till now we are still at the classical realm. By imposing equal time canonical commutations on the vector potentials, the Fourier expansion coefficients become operators satisfying the commutation relations

$$[a_r(\mathbf{k}), a_s^*(\mathbf{k}')] = \zeta_r \delta_{rs} \delta_{\mathbf{k}\mathbf{k}'} \quad (2.12)$$

$$[a_r(\mathbf{k}), a_s(\mathbf{k}')] = [a_r^*(\mathbf{k}), a_s^*(\mathbf{k}')] = 0 \quad (2.13)$$

where $\zeta_r = 1$ for $r = 1, 2, 3$ and $\zeta_0 = -1$ for $r = 0$. Following the Gupta-Bluerer approach (Schweber 1961, 245-251), $a_r^*(\mathbf{k})$ are taken to be creation operators and $a_r(\mathbf{k})$ as absorption operators, even if in the case $r = 0$ their role seems to be interchanged due to the minus signal sign. This would mean that $A^0(x)$ is an anti-Hermitian operator. In the Gupta-Bluerer method this problem thus not arise due to the use of an indefinite metric.

The number operators are defined as $N_r(\mathbf{k}) = \zeta_r a_r^*(\mathbf{k}) a_r(\mathbf{k})$, implying that the total energy operator is given by

$$H = \sum_{\mathbf{k}} \sum_{r=0}^3 \hbar \omega_{\mathbf{k}} \xi_r a_r^*(\mathbf{k}) a_r(\mathbf{k}) \quad (2.14)$$

With this choice of absorption and creation operators, and corresponding definition of number and energy operators there is a consistent interpretation of these operators, because we do not have any negative number of photons or energy appearing (due to the scalar photons). However there would be a problem of having states of the quantized field with negative norm. Still we must recall that we are taking the creation and absorption operators to be independent, and that cannot be the case, since we must take into account the Lorentz condition. In the Gupta-Bluerer method it is made use of a Lorentz condition ($\partial_\mu A^{\mu+}(x)\psi = 0$) which is less stringent than its classical counterpart. This is necessary to have no contradiction with the commutation relations. The subsidiary condition selects the physically realizable states, all with a positive-defined norm, in which we have $[a_3(\mathbf{k}) - a_0(\mathbf{k})]\psi = 0$. This implies that the physical states have an admixture of longitudinal and scalar photons. As a result of this constraint regarding longitudinal and scalar photons, all observable quantities of the field in free space will depend only on the transverse photons. For example the expectation value of the energy is given by

$$\langle \psi | H | \psi \rangle = \langle \psi | \sum_{\mathbf{k}} \sum_{r=1}^2 \hbar \omega_{\mathbf{k}} \xi_r a_r^*(\mathbf{k}) a_r(\mathbf{k}) | \psi \rangle \quad (2.15)$$

This does not mean that the longitudinal and scalar photos are irrelevant. In reality they cannot be seen as independent dynamical degrees of freedom of the field as the transverse part is. When considering for example the electron-electron scattering the longitudinal and scalar photons represent (in a covariant way) the Coulomb interaction between the electrons (Landau and Lifshitz 1971, 124-125; Björken and Drell 1965, 78-81).

With this approach it is guaranteed that the Hamiltonian (energy) operator cannot have negative values. This means that we have a lower bound to the energy of the quantized electromagnetic field. Since in the admissible physical states the contribution to the total energy and momentum of the field due to the time-like and longitudinal photons cancels, the ground state corresponds to a state with the lowest contribution from the transverse photons, that is, it is a state that does not contain any transverse photons. This does not impose any restriction on the number of time-like and longitudinal photons in this ground state (besides the one provided by the subsidiary condition that implies there is an equal number of them in the allowed states). However states with different admixtures of time-like and longitudinal photons correspond to a particular choice within the Lorentz gauge, since the Lorentz condition does not specify the potential uniquely (Mandl and Shaw 1984, 89). This means that there really is no physical difference between ‘different’ ground states with different admixtures of non-transverse photons (Schweber 1961, 251; Källén 1972, 42). In this way, we may simply characterize the ground state (without choosing a particular Lorentz gauge) by requiring the occupation number for the transverse photons to be zero (Jauch and Rohrlich 1976, 47). The classical counterpart of this ground state is simply the space vacuum: a region of space without any electromagnetic field.

3 From the quantum electrodynamical vacuum to the quantum electromagnetic vacuum

The more basic and fundamental elements of quantum electrodynamics are already present in Paul Dirac's 1927 non-relativistic treatment of the interaction between a quantized radiation field and an atomic system (Dirac 1927). In it, initially, the electromagnetic field and matter are described by classical Hamiltonians. A further term gives the interaction between the field and matter. All this can be developed within a correspondence approach with classical mechanics and field theory, that is, this type of Hamiltonian can be put to use in the Maxwell-Lorentz classical electrodynamics or a classical theory of fields in interaction (Barut 1964, 138; Bogoliubov and Shirkov 1959, 84). Then a second 'layer' is put on top of the classical description through which the quantization of the individual fields is achieved. That is, the generalized coordinates (and conjugate momenta) of each field are submitted to commutation or anticommutation relations, and the terms in the Hamiltonian for each field become operators, as is also the case for the term describing the interaction between the fields. But it is important to notice that the fields are quantized as free non-interacting fields, each by itself. Then we are into the game. For practical purposes Dirac makes use of perturbation theory to treat the interaction of radiation and matter.² So it was then, and it still is now.

3.1 The Divergence of the S-matrix series expansion and the concept of vacuum state

For the purpose of this paper it will not be necessary to address the quantum electrodynamical treatment of the electromagnetic field in interaction with the electron-positron field. It will suffice to consider the quantum electrodynamical description of the interaction of the electromagnetic field with a linear dipole oscillator. However there is one result that is fundamental for the ideas to be proposed. In quantum electrodynamics, the majority of its applications are made using the S-matrix formalism. This formalism is particularly tailor-made for the description of scattering processes but is also applicable to bound-state problems (Veltman 1994, 62-67). I follow Dyson's presentation of a typical scattering process as described within quantum electrodynamics:

The free particles which are specified by a state A in the remote past, converge and interact, and other free particles emerge or are created in the interaction and finally constitute the state B in the remote future. (Dyson 1952a, 81)

Dyson calls attention to the fact that:

² The use of perturbative methods has a long history in celestial mechanics. One example is the development of an analytical perturbation theory for the three-body problem: the Sun-Earth-Moon system (Hoskin and Taton 1995, 89-107). From the planets, perturbative methods went to the planetary models of atoms, being a calculational tool present in the so-called old quantum theory (Darrigol 1992, 129 and 171). Also it became fundamental in the creation of matrix mechanics, as it was from the perturbative study of the anharmonic oscillator that Werner Heisenberg developed his quantum-theoretical approach (Darrigol 1992, 266-267; Paul 2007, 4-5). Soon after, Heisenberg and Max Born put together a perturbation theory within the formalism of quantum mechanics recently developed (van der Waerden 1967, 43-50; see also Lacki 1998).

The unperturbed states A and B are supposed to be states of free particles without interaction and are therefore represented by constant state-vector ϕ_A and ϕ_B in the interaction representation. The actual initial and final states in a scattering problem will consist of particles each having a self-field with which it continues to interact even in the remote future and past, hence ϕ_A and ϕ_B do not accurately represent the initial and final states. (Dyson 1952a, 81)

Dyson presents what can be considered an operational justification for using the states of free particles (usually referred to as bare states) in the calculations, by taking into account how scattering experiments are really done (see also Falkenburg 2007, 129-131):

In an actual scattering experiment the particles in state B are observed in counters of photographic plates or cloud-chambers and the time of their arrival is not measured precisely. Therefore it is convenient to use for B not the state-function $\Psi_B(t)$ but a state function ϕ_B which is *by definition* the state-function describing a set of bare particles without radiation interaction [(that is without self-interaction with its own field)], the bare particles having the same momenta, and spins as the real particles in state B. (Dyson 1952a, 94)

The transition amplitude of the scattering process is given by $S_{AB} = (\phi_B^* S \phi_A)$, where S is the so-called S-matrix.

One of the major achievements of Dyson in the development of quantum electrodynamics was showing that the perturbative expansion of the S-matrix is renormalizable to all orders. Quantum electrodynamics (QED) had tremendous problems of divergent integrals that made impossible but a few lower order calculations (Pais 1986, 374-376). This problem was circumvented by the procedure of mass and charge renormalization. Dyson showed, in a paper published in 1949, that the renormalization procedure could be applied to all orders of the perturbative expansion of the S-matrix in power-series of e^2 , where e is the electric charge (Schweber 1994, 527-544). Dyson even considered that “all QED was the perturbative series” (quoted in Schweber 1994, 565). This view by Dyson has been vindicated by Damiano Anselmi, how wrote that

To describe the interaction between quanta it is compulsory to proceed *perturbatively*.

Perturbation theory is the iterative procedure by which the elementary interactions are composed into complex interactions. Perturbation theory is not just a tool for making calculations: it is the very same *formulation* of quantum field theory. In classical mechanics well-defined differential equations describe the evolution of a system subject to forces of various types. Solving those equations is in general very difficult. *Therefore* we approximate. In quantum mechanics the situation is similar, given that the forces are anyway external, therefore classical. Still one has well-defined equations, which can be treated with the method of approximations when the exact solutions are not available. In quantum field theory, however, writing the equations themselves is difficult, not only solving them. So . . . we approximate.

Approximate *what*? One understands the idea of approximation in a *relative* sense, when the exact solution is inaccessible, but can be approached with arbitrary precision. In quantum field theory the idea of approximation becomes *absolute*: we just approximate, we do not approximate *something*. (Anselmi 2003, 311)

The possible problem with this view is that in the summer of 1951, soon after he presented his S-matrix formulation of quantum electrodynamics, Dyson came out with a physical argument that strongly suggested that, after all, “all the power-series expansions currently in use in quantum electrodynamics are divergent after the renormalization of mass and charge” (Dyson 1952b, 631). According to Dyson the

series could at best be an asymptotic expansion. This situation led Dyson to consider that “you didn’t really have a theory” (quoted in Schweber 1994, 565).³

I consider that this drastic conclusion by Dyson can be avoided, by following Dyson’s own intuition regarding quantum electrodynamics. The fact we are facing in quantum electrodynamics is that it is not possible to treat radiation and matter as one closed system. This results from the type of equations we have in the theory:

these equations [(the coupled Maxwell-Lorentz and Dirac equations)] are non-linear. And so there is no possibility of finding the general commutation rules of the field operators in closed form. We cannot find any solution of the field equations, except for the solutions which are obtained as formal power series expansions in the coefficient e which multiplies the non-linear interaction terms. It is thus a basic limitation of the theory, that it is in its nature a perturbation theory starting from the non-interacting fields as an unperturbed system. Even to write down the general commutation laws of the fields, it is necessary to use perturbation theory of this kind. (Dyson 1952a, 79)

In my view, this indicates that Anselmi’s argument is still valid when accepting that the power series expansion of the S-matrix, can “only be an asymptotic series” (Schweber 1994, 565). The type of equations of the theory imposes the need for a perturbative approach. Now, the fact that we only have an asymptotic series means that there is no ‘exact’ solution (even if just as a power series). This gives a stronger argument for Anselmi’s view that “in quantum field theory the idea of approximation becomes *absolute*: we just approximate, we do not approximate *something*” (Anselmi 2003, 311), and that “perturbation theory is not just a tool for making calculations: it is the very same *formulation* of quantum field theory” (Anselmi 2003, 311). It is because of this, that, contrary to Dyson’s conclusion, I still feel that quantum electrodynamics is the perturbative series, as Dyson originally defended (and as Anselmi defends).⁴

This *de facto* situation has immediate consequences regarding the interpretation of the mathematical formalism of the theory. The situation we are facing is that even if looking solely into the mathematical formalism of the theory we can talk about the Hilbert space of the physical states of the full Hamiltonian of the two fields and their interaction, in quantum electrodynamics we cannot build these interacting states. The interacting states would be constructed (by an iteration procedure) from the Fock states of each field (Schweber 1961, 322). Since the S-matrix series expansion is divergent we know that we cannot obtain these interacting states (Scharf 1995, 314-318). This implies in particular that there is no physical meaning within quantum electrodynamics to the ground state of the interacting fields. It is usually thought that the coupled fields vacuum state can be “formally expanded as a superposition of ϕ_0 ” (Redhead 1982, 86; Schweber 1961, 655), where ϕ_0 are the vacuum states of the free fields. This is not possible to do in quantum electrodynamics. However, this does not imply that the concept of vacuum is not relevant in the theory, as a closer look at the quantized electromagnetic field reveals (I will not consider here the ground state of the electron-positron field).

3.2 The Casimir effect and formal aspects related to the vacuum state

³ Even if strict mathematical proof of the divergence of the S-matrix does not exist, further strong evidence in favour of Dyson’s claim has been given in the last decades. (Aramaki 1989, 91-92; West, 2000, 180-181; Jentschura, 2004, 86-112; Caliceti et al. 2007, 5-6)

⁴ This brings the philosophical question of what to make of a ‘theory’ that seems to provide only an approximate scheme for calculations. As Meinard Kuhlmann remarked (in a more broader context), quantum electrodynamics seems more like “a set of formal strategies and mathematical tools than a closed theory” (Kuhlmann 2006). This is an important question, but it would go beyond the scope of this paper to address it here.

Contrary to the classical case, the ground state of the (free) quantized electromagnetic field is presented as having quite a few very ‘visible’ physical effects, in particular the so-called Casimir effect. In the usual interpretation of the Casimir effect the ground state of the electromagnetic field could have a dynamical effect on macroscopic conducting plates located face to face in the form of an attractive force between the plates. Another so-called (electromagnetic) vacuum effect would be the spontaneous emission of radiation by atoms in an excited state without radiation present (Aitchison 1985, 342-345; Milonni 1994, 79-111). A different physical interpretation can be given to these (and other) so-called vacuum effects without explicit resort to the ground state of the field (e.g. Milonni 1994, 115-138; Zinkernagel 1998, 48-60). I will look only into the case of the Casimir effect.

When considering the quantization of the electromagnetic field (or even a simple harmonic oscillator) the canonical quantization procedure does not enforce a specific choice of the ordering of the creation and absorption operators appearing in the field operators, energy operators, momentum operators, etc. Following Dirac (1958, 84-88) we may recall that in classical mechanics a dynamical system can be described in terms of generalized coordinates q_j and momenta p_j . Let u and v be two dynamical variable functions of q_j and p_j . The Poisson bracket of these two functions is

$$[u, v] = \sum_r \left\{ \frac{\partial u}{\partial q_r} \frac{\partial v}{\partial p_r} - \frac{\partial u}{\partial p_r} \frac{\partial v}{\partial q_r} \right\} \quad (3.1)$$

In the canonical quantization procedure a quantum equivalent of the Poisson bracket is introduced

$$uv - vu = i\hbar[u, v] \quad (3.2)$$

where u and v are now taken to be operators. This method is the one used in the case of the quantization of the electromagnetic field. Looking into the case of the quantization of one field mode (mathematically equivalent to the quantization of the harmonic oscillator), in terms of creation (a^*) and absorption (a) operators, we have

$$H = \frac{\hbar\omega}{2}(aa^* + a^*a) \quad (3.3)$$

Since $[a, a^*] = 1$, we can write the previous expression as

$$H = \hbar\omega \left(a^*a + \frac{1}{2} \right) \quad (3.4)$$

In here we see a term $\hbar\omega/2$ corresponding to the so-called zero-point energy of the field mode. This would imply that the electromagnetic field when in its vacuum state would have an energy different from zero (formally infinite). This zero-point energy can be dealt with by recalling (with Dirac) that energy measurements are made in relation to the ground state energy, which enable us to set it to zero:

$$H - \langle 0|H|0 \rangle = \frac{1}{2}\hbar\omega(aa^* + a^*a) - \frac{1}{2}\hbar\omega = \frac{1}{2}\hbar\omega(2a^*a + 1) - \frac{1}{2}\hbar\omega = \hbar\omega a^*a \quad (3.5)$$

A different way of addressing this question is to take away any physical meaning to it and to see the term $\hbar\omega/2$ as resulting simply from an imprecision in the quantization procedure (e.g. Teller 1995, 129-131). In fact at a classical level there is no difference between $\hbar\omega/2(aa^* + a^*a)$ and $\hbar\omega a^*a$. This means that there is an ambiguity in the ordering of the operators, since depending on our starting classical expression we obtain a different quantum Hamiltonian. In this way we can consider the zero-point energy as an artifact of an improper application of the quantization rule, and use the so-called normal ordering in which we have the operator $H = \hbar\omega a^*a$, where there is no zero-point energy (this is the ordering adopted in the previous section). Contrary to this view, Hendrik Casimir presented in 1948 a calculation sustaining that there were dynamical consequences of this zero-point energy (Casimir 1948).

In the quantization of the electromagnetic field it is (for practical purposes) considered the space to be divided in ‘boxes’ with a volume $V = L^3$, and to impose on the field the periodic boundary conditions $\mathbf{A}(x + L, y + L, z + L, t) = \mathbf{A}(x, y, z, t)$. This implies (since $\mathbf{A} \sim \exp i\mathbf{k}\cdot\mathbf{r}$) that $(k_x, k_y, k_z) = 2\pi/L (l, m, n)$, where l, m, n are integers. It is considered that “this artificial periodic boundary condition will be of no physical consequences if L is very large compared with any physical dimensions of interest” (Milonni 1994, 44). If we take there to be two (infinite) parallel conducting plates located, say, at $z = 0$ and $Z = d$, Casimir made the heuristic move of considering that this changed the set of modes describing the quantized electromagnetic field who would not be anymore simply given by free-space plane-wave modes. This would mean that matter would affect radiation by simply changing its quantization boundary conditions. According to Casimir since we are considering perfect conductors, the tangential component of the electric field must vanish on the walls of the conductors. This implies that

$$k_x = \frac{l\pi}{L}, k_y = \frac{m\pi}{L}, k_z = \frac{n\pi}{d} \quad (3.6)$$

In this way the allowed frequencies will be

$$\omega_{lmn} = k_{lmn} c = \pi c \left[\frac{l^2}{L^2} + \frac{m^2}{L^2} + \frac{n^2}{d^2} \right]^{1/2} \quad (3.7)$$

The zero-point energy is given by

$$\sum_{l,m,n} \frac{1}{2} \hbar \omega_{lmn} = \sum_{l,m,n} \pi \hbar c \left[\frac{l^2}{L^2} + \frac{m^2}{L^2} + \frac{n^2}{d^2} \right]^{1/2} \quad (3.9)$$

(in the summation a factor of 2 must be considered due to the polarization, when $l, m, n \neq 0$). The difference of the zero-point energy of the field with and without plates $U(d) = E(d) - E(\infty)$ is taken to be the potential energy of the system plates + vacuum field. That is $U(d)$ is now taken to be the energy required to bring the plates from a large distance to d . From the previous expression it is derived the expression for the force per unit area $F(d) = -U'(d)$ between the plates:

$$F(d) = -\frac{\pi^2 \hbar c}{240 d^4} \quad (3.10)$$

This is, we might say, the conventional derivation of the Casimir effect as a vacuum effect. However, as Peter W. Milloni and Mei-Li Shih(1992) have shown, it is possible to arrive at the Casimir force between the plates without having to consider any vacuum field in the calculation. This is done simply by adopting the normal ordering of the operators and by taking explicitly into account that the conducting plates are not mathematical boundary conditions but must be considered as constituted by matter. We will consider two semi-infinite dielectric slabs with dielectric constants ϵ_1 and ϵ_2 at a distance d separated by a layer with dielectric constant ϵ_3 . To derive the Casimir result it will be considered the limiting case of a perfect conductor. There is an induced dipole moment in each atom of the slabs induced by source fields. In the Milonni and Shih calculation the approximation was made that “each dipole interacts, in effect, only with its own field; this field is modified from its free-space form by the presence of all the other dipoles” (Milonni and Shih 1992, 4245). The energy of this system (of two dielectric slabs separated by a medium) is given by

$$\langle E \rangle = -\frac{1}{2} \int d^3 r \langle \mathbf{P}(\mathbf{r}, t) \cdot \mathbf{E}(\mathbf{r}, t) \rangle \quad (3.11)$$

where $\mathbf{P}(\mathbf{r}, t)$ is the polarization due to the dipoles and $\mathbf{E}(\mathbf{r}, t)$ is the electric field present. The electric field can be written as $\mathbf{E}(\mathbf{r}, t) = \mathbf{E}_0(\mathbf{r}, t) + \mathbf{E}_s(\mathbf{r}, t)$, where $\mathbf{E}_0(\mathbf{r}, t)$ is the source-free (or vacuum) part of the electric field and $\mathbf{E}_s(\mathbf{r}, t)$ is the part due to the dipoles present. By using a normal ordering of the field operators we have

$$\langle E \rangle = -\frac{1}{2} \int d^3 r \langle \mathbf{P}(\mathbf{r}, t) \cdot \mathbf{E}_s(\mathbf{r}, t) \rangle \quad (3.12)$$

in which we have only source fields. Milonni and Shih then calculated the change in the energy due to an infinitesimal change δd in d , and from this the force between the dielectric slabs. The general expression is

$$F(d) = -\frac{\hbar}{2\pi^2 c^3} \int_1^\infty dp p^2 \int_0^\infty d\xi \xi^3 \epsilon_3^{3/2} \left[\left[\frac{\epsilon_3 s_1 + \epsilon_3 p}{\epsilon_3 s_1 - \epsilon_1 p} \frac{\epsilon_3 s_2 + \epsilon_2 p}{\epsilon_3 s_2 - \epsilon_2 p} e^{2\xi p \sqrt{\epsilon_3} d/c} - 1 \right]^{-1} + \left[\frac{s_1 + p}{s_1 - p} \frac{s_2 + p}{s_2 - p} e^{2\xi p \sqrt{\epsilon_3} d/c} - 1 \right]^{-1} \right] \quad (3.13)$$

I will not go into the details of this expression, but simply consider what force this approach predicts in the same case as the one considered by Casimir. We have empty space between the slabs, which means that $\epsilon_3 = 1$; also we must consider the case of two perfectly conducting plates, which implies taking $\epsilon_{1,2} \rightarrow \infty$. Under these conditions the previous expression reduces to

$$F(d) = -\frac{\pi^2 \hbar c}{240d^4} \quad (3.14)$$

which is the Casimir force. In this way, Milonni and Shih shown that “the Casimir effect can be understood in terms of source fields in conventional quantum electrodynamics, with no explicit reference to the zero-point energy” (Milonni and Shih 1992, 4241).

A different view was given by Simon Saunders, who wrote: “I do not think we can do without appeal to the zero-point energy in explaining the Casimir effect” (Saunders 2002, 23). Saunders develops his argumentation without taking into account Milonni’s work. He mentions for example Lifshitz macroscopic theory, concluding that “Lifshitz’s methods are perfectly consistent with the interpretation of the effect in terms of vacuum fluctuations” (Saunders 2002, 19). This is highly doubtful. As we have seen, according to Milonni & Shih, the Casimir force can “be calculated in terms of source fields, with no explicit reference to zero-point-field energy” (Milonni and Shih 1992, 4241). Also they show that “the general Lifshitz expression, and therefore the Casimir force in particular, may be derived in terms of sources alone in conventional QED” (Milonni and Shih 1992, 4243).

Even when agreeing with Milonni’s interpretation, this does not mean that the ground state of the quantized electromagnetic field becomes a sort of ‘nothingness’ without any physical relevance, as it is the case when we consider the classical electromagnetic vacuum. That is not the case. As Milonni has remarked, “the vacuum field is absolutely necessary in the quantum theory of radiation, if only to preserve commutation relations and the formal consistency of the theory” (Milonni 1994, 138). In the classical case we can conceive a sole dipole in empty space. In this case “the only field acting on the dipole is its own radiation reaction field” (Milonni 1994, 52). The difference from the classical case is that when considering the quantized electromagnetic field “there is an ‘external’ field, namely, the source-free or vacuum field” (Milonni 1994, 52). Milonni shows that, in contrast to the classical case, the ground state of the free electromagnetic field cannot be disregarded as soon as a charged body (which can be seen as a source of electromagnetic field) is considered, as it is necessary to take into account the source-free field variables for the preservation of the commutation relations (Milonni 1994, 53).

Let us look at this in more detail. The Hamiltonian for the dipole oscillator in interaction with the quantized electromagnetic field can be written as

$$H = \frac{1}{2} \left(\mathbf{p} - \frac{e}{c} \mathbf{A} \right)^2 + \frac{1}{2} m \omega_0^2 \mathbf{x}^2 + H_F \quad (3.15)$$

where H_F is the field Hamiltonian, \mathbf{x} is the operator corresponding to the classical coordinate of the oscillator, \mathbf{p} is the operator for the dipole momentum, and ω_0 is the frequency of oscillation of the dipole. In the Heisenberg representation we have (in the electric dipole approximation):

$$\ddot{\mathbf{x}} + \omega_0^2 \mathbf{x} = \frac{e}{m} \mathbf{E} = i \frac{e}{m} \sum_{\mathbf{k}\lambda} \left(\frac{2\pi\hbar\omega_k}{V} \right)^{1/2} \left[a_{\mathbf{k}\lambda}(t) - a_{\mathbf{k}\lambda}^*(t) \right] \mathbf{e}_{\mathbf{k}\lambda} \quad (3.16)$$

$$\dot{a}_{\mathbf{k}\lambda} = a_{\mathbf{k}\lambda}(0)e^{-i\omega_{\mathbf{k}}t} + ie\left(\frac{2\pi}{\hbar\omega_{\mathbf{k}}V}\right)^{1/2} \int_0^t dt' \mathbf{e}_{\mathbf{k}\lambda} \cdot \dot{\mathbf{x}}(t')e^{i\omega_{\mathbf{k}}(t'-t)} \quad (3.17)$$

(where $a_{\mathbf{k}\lambda}$ and $a_{\mathbf{k}\lambda}^*$ are respectively the photon annihilation and creation operators for the field mode (\mathbf{k}, λ) , $\mathbf{e}_{\mathbf{k}\lambda}$ are the polarization vectors, and \mathbf{E} is the electric field operator). In this way the Heisenberg equation for the operator \mathbf{x} can be written as:

$$\ddot{\mathbf{x}} + \omega_0^2 \mathbf{x} = \frac{e}{m} \mathbf{E}_0(t) + \frac{e}{m} \mathbf{E}_{\text{RR}}(t) \quad (3.18)$$

where we have

$$\mathbf{E}_0(t) = i \sum_{\mathbf{k}\lambda} \left(\frac{2\pi\hbar\omega_{\mathbf{k}}}{V} \right)^{1/2} \left[a_{\mathbf{k}\lambda}(0)e^{-i\omega_{\mathbf{k}}t} - a_{\mathbf{k}\lambda}^*(0)e^{i\omega_{\mathbf{k}}t} \right] \mathbf{e}_{\mathbf{k}\lambda} \quad (3.20)$$

$$\mathbf{E}_{\text{RR}}(t) = -\frac{4\pi e}{V} \sum_{\mathbf{k}\lambda} \int_0^t dt' [\mathbf{e}_{\mathbf{k}\lambda} \cdot \dot{\mathbf{x}}(t')] \mathbf{e}_{\mathbf{k}\lambda} \cos \omega_{\mathbf{k}}(t'-t) = \frac{2e}{3c^3} \ddot{\mathbf{x}} \quad (3.21)$$

According to Milonni, “ $\mathbf{E}_0(t)$ is the free or zero-point field acting on the dipole. It is the homogeneous solution of the Maxwell-Lorentz equation for the field acting on the dipole, i.e. the solution, at the position of the dipole, of the wave equation $[\nabla^2 - c^{-2} \partial^2/\partial t^2]\mathbf{E} = 0$ satisfied by the field in the (source-free) vacuum. For this reason $\mathbf{E}_0(t)$ is often referred to as the *vacuum field* ... $\mathbf{E}_{\text{RR}}(t)$ is the source field, the field generated by the dipole and acting *on* the dipole” (Milonni 1994, 52).

Considering equation 3.18 that can be written as

$$\ddot{\mathbf{x}} + \omega_0^2 \mathbf{x} - \tau \ddot{\mathbf{x}} = \frac{e}{m} \mathbf{E}_0(t) \quad (3.22)$$

(where $\mathbf{E}_0(t)$ is, as mentioned, the vacuum electric field operator and $\tau = 2e^2/3mc^3$), and the corresponding equation for the momentum operator \mathbf{p} , we have the following commutation relation (in the z-direction) between the two operators

$$[z(t), p_z(t)] \cong \frac{2ei\hbar e^2}{3\pi mc^3} \omega_0^3 \int_{-\infty}^{\infty} \frac{dx}{x^2 + \tau^2 \omega_0^6} = \frac{2ei\hbar e^2 \omega_0^3}{3\pi mc^3} \frac{\pi}{\tau \omega_0^3} = i\hbar \quad (3.23)$$

as is expected according to general quantum mechanics rules. Now if we had not considered the vacuum field $\mathbf{E}_0(t)$, then in the operator equation for \mathbf{x} , “the operator $\mathbf{x}(t)$ would be exponentially damped, and commutators like $[z(t), p_z(t)]$ would approach zero for $t \gg (\tau\omega_0^2)^{-1}$ ” (Milonni, 1994, p. 53). Because of this Milonni concludes that “the free field is in fact *necessary* for the formal consistency of the theory” (Milonni 1994, 53).

This result and Milonni’s derivation of the Casimir effect in the context of standard quantum electrodynamics has been questioned. In Rugh, Zinkernagel and Cao (1999, 129) two critical remarks are made on Milonni’s (and his collaborators) approach. One point is that Milonni’s approach regarding the derivation of the Casimir effect as due

only to source fields is not conclusive, because in higher orders of perturbative calculations we will have contributions to the vacuum energy from the so-called vacuum blob diagrams. But this is the case only when considering the ‘interacting’ vacuum (e.g. Rugh and Zinkernagel 2002, 675), it has nothing to do with the quantized source-free electromagnetic field even if in its vacuum or ground state. The other point, also made in Rugh and Zinkernagel (2002, 683, footnote 50), is that Milonni uses the so-called fluctuation-dissipation theorem for linearly dissipative systems to arrive at his result that the source-free field is necessary for consistency reasons (i.e. the preservation of commutation relations), and this is not a sound approach. That is not the case, even if Milonni presents his results as an example of the theorem (e.g. Milonni 1988, 106). The need for the source-free field for the preservation of the commutation relations follows simply from the formalism of quantum electrodynamics (Milonni 1984, 342; Milonni 1988, 106; Milonni 1994, 50-54). That is, even if the interaction of radiation and matter can be treated only with approximate procedures, in all cases, we must have simultaneously with a system consisting in charged matter a different system corresponding to the quantized free electromagnetic field, *even if only in its ground state (when considering charged matter in empty space)*.

3.3 The physical meaning of variance

We can consider that the necessity of taking into account the ground state of the quantized electromagnetic field goes beyond its more formal aspects. As is well known, in the ground state of the quantized electromagnetic field the expectation value of the electric and magnetic fields vanishes (that is $\langle 0|\mathbf{E}|0\rangle = \langle 0|\mathbf{B}|0\rangle = 0$), but not its variance since $\langle 0|\mathbf{E}^2|0\rangle$ and $\langle 0|\mathbf{B}^2|0\rangle$ are non-zero in the ground state. What to make of this result? There is a tendency in the literature to refer to the non-vanishing variance as ‘fluctuations’ of the vacuum state (Sakurai 1967, 32-33; Aitchison 1985, 246-247). For example, according to I. J. R. Aitchison “the vacuum can now be thought of as a state in which the fields are all in their ground states, but executing random fluctuations (even at $T = 0$) about their zero average values” (Aitchison 1985, 347). We must take some care in adopting this type of terminology. As has been mentioned, we cannot associate the non-vanishing variance of the ground state of the quantized electromagnetic field to some sort of fluctuation in time: “there is no time evolution of this vacuum state” (Rugh and Zinkernagel 2002, 673). I will defend here that the mathematical result of a non-vanishing variance of the quantized electromagnetic field can be given an interpretation that has a clear experimental meaning.

According to the interpretation of quantum mechanics adopted here, the non-zero variance (or its square root, the standard deviation) is determined by considering a large (ideally infinite) number of measurements performed in similarly prepared systems (Isham 1995, 80-81; Peres 1995, 24-26; Ballentine 1998, 225-227, Falkenburg 2007, 205-207). According to what Christopher J. Isham called the minimal interpretation of quantum theory:

Quantum theory is viewed as a scheme for predicting the probabilistic distribution of the outcomes of *measurements* made on suitably prepared copies of a system.

The probabilities are interpreted in a statistical way as referring to the *relative frequencies* with which various results are obtained if the measurements are repeated a sufficiently large number of times. (Isham 1995, 80)

Asher Peres's view is that

A *quantum system* is a useful abstraction ... defined by an *equivalent class of preparations*. For example there are many equivalent macroscopic procedures for producing what we call a photon, or a free hydrogen atom, etc. The *equivalence* of different preparation procedures should be verifiable by suitable tests.... While quantum systems are somewhat elusive, quantum *states* can be given a clear *operational* definition, based on the notion of test. Consider a given preparation and a set of tests ... if these tests are performed many times, after identical preparations, we find that the statistical distribution of outcomes of each test tends to a limit. Each outcome has a definite probability. We can then define a state as follows: *A state is characterized by the probabilities of the various outcomes of every conceivable test....* Before we examine concrete examples, the notion of *probability* should be clarified. It means the following. We *imagine* that the test is performed an infinite number of times, on an infinite number of replicas of our quantum system, all identically prepared. This infinite set of experiments is called a *statistical ensemble*. ... In this statistical ensemble, the occurrence of event A has relative frequency $P\{A\}$; it is this relative frequency which is called *probability*. (Peres 1995, 24-25)

Under this view we cannot associate for example the Schrödinger wave function to one sole system. We must consider a large number of identical 'quantum systems' prepared in the same way and then subjected to the same measurement procedure. From the wave function we can calculate the relative frequencies (probabilities) of particular outcomes of experiments, e.g. of scattering events of a given type.

As is well known the wave packet collapse is a central aspect of the so-called Copenhagen interpretation of quantum mechanics. In his "Who invented the 'Copenhagen interpretation'? A study in mythology", Don Howard (2004) calls the attention to the fact that Bohr's interpretation, "makes no mention of wave packet collapse or any of the other silliness that follows therefrom, such as a privileged role for the subjective consciousness of the observer" (Howard 2004, 669). This view had already been anticipated by Paul Teller in his "The projection postulate and Bohr's interpretation of quantum mechanics". Teller starts with the question: "Why does Bohr nowhere discuss the projection postulate?" (Teller 1980, 201). His answer is:

My position is very simply that Bohr gives the state function a statistical interpretation and a statistical interpretation has no need of the projection postulate [i.e. the collapse of the wave function during measurement]. (Teller 1980, 211)

Teller's view is that to Bohr

The state function must be taken as a purely symbolic device for calculating the statistics of classically or commonly described experimental outcomes in collections of phenomena grouped by shared specifications of experimental conditions. (Teller 1980, 206)

Accordingly,

On Bohr's view the state function describes not one individual case, but a whole ensemble of cases with a common preparation characterized in the language of classical physics or daily discourse. (Teller 1980, 213)

I agree with Teller's reading or 'interpretation' of Bohr. This means that what we nowadays call the statistical or ensemble interpretation (i.e. Isham's minimal interpretation) was basically the interpretation of Bohr, or at least we can see it as an interpretation compatible with what we know about Bohr's views on quantum theory, i.e. a Bohrian interpretation. It is this Bohrian interpretation that is being followed in this work.

Now we can address the meaning of having $\langle 0|E^2|0\rangle \neq 0$. We must consider a particular experimental setup that will enable us to make measurements on a quantum electromagnetic field. We can consider successive measurements made on a field, which can be taken to be in the same (initial) state at each successive measurement, or we may think in terms of different identical experimental setups being used at the same time to make measurements on identically prepared fields (in this case all in the vacuum state). Then on each of a large number of similarly prepared systems a measurement is made on the electric field. According to the interpretation of the theory, there will be a statistical spread (distribution) in the results of independent measurements according to a standard deviation of $\sqrt{\langle 0|E^2|0\rangle}$ from the value corresponding to no field. That is, it will be measured sometimes an electric field different from zero even if the electromagnetic field is in its ground state. Is this again a formal aspect of the theory, even if a formal aspect of its interpretation? Or are there any real experiments where a measurement is made on the vacuum field and results are obtained corresponding to a statistical distribution of results deviating from ‘nothingness’?

3.4 Experimental results on the vacuum state using the balanced homodyne detection method

In experiments using the method known as balanced homodyne detection it is possible to determine what can be interpreted as quadrature fluctuations of the vacuum or simply vacuum noise that corresponds to the non-zero standard deviation predicted by the theory (Leonhardt 1997, 23, 47 and 84-88).

In the balanced homodyne method, the signal (under study) is sent into one of the ports of a beam splitter. In the present case since we are making a measurement of the vacuum electromagnetic field, the port is left unused, that is, there is no external field present. The other port receives a strong coherent laser field (called the local oscillator), which will provide the phase reference for measuring the quadrature statistics of the signal field, which in this case is the vacuum.

We can write the quantum field operator E_x for a single-mode field (assumed to be polarized along the x-direction) as

$$E_x = \left(\frac{\hbar\omega}{\epsilon_0 V} \right)^{1/2} (a e^{-i\omega t} + a^* e^{i\omega t}) \sin(kz) \quad (3.24)$$

where ω is the frequency of the mode and k is the wave number related to the frequency according to $k = \omega/c$. Defining the so-called quadrature operators

$$q = \frac{1}{\sqrt{2}}(a^* + a) \quad (3.25)$$

$$p = \frac{i}{\sqrt{2}}(a^* - a) \quad (3.26)$$

the field operator can be written as

$$E_x = \sqrt{2}\epsilon_0 \sin(kz)(q \cos(\omega t) + p \sin(\omega t)) \quad (3.27)$$

The balanced homodyne detection makes it possible to make a measurement of these quadrature components of a quantum field.

The beam splitter will combine the incoming fields. Taking, for simplicity, each incoming field to be described by the mode operators a and b , the output modes are

$$c = \frac{1}{\sqrt{2}}(a + ib) \quad (3.28)$$

$$d = \frac{1}{\sqrt{2}}(b + ia) \quad (3.29)$$

After this optical mixing of the signal and the local oscillator, each beam is directed towards a photodetector, which enables a measurement of the field intensities. The photodetectors responds to the intensity of the incident light, generating the photocurrents $I_c = \langle c^* c \rangle$ and $I_d = \langle d^* d \rangle$. By assuming that the photocurrents are proportional to the photon numbers n_c and n_d of the beams striking each detector, we have that the difference $I = I_c - I_d$ is proportional to the difference photon number

$$n = n_c - n_d = 2^{1/2} |\alpha| q_\theta \quad (3.30)$$

where $|\alpha|^2$ is the intensity of the coherent field (local oscillator), and q_θ is a quadrature component of the vacuum field (signal). By changing the phase of the coherent field it is possible to measure an arbitrary quadrature of the signal field. In particular if the phase was initially chosen so to measure q by changing the phase by $\pi/2$ it is possible to measure p ; in this way a balanced homodyne detector enables to measure the quadrature components defining the quantum state of the field (Gerry and Knight 2005, 167-168). We have then an experimental procedure that makes possible the experimental study of quantum states of light, like the vacuum state.

Let us see in more detail what the experiment tells us about the vacuum state. First we must recall the interpretation of the quantum formalism. If we use the balanced homodyne detector to make one measurement of a quadrature component that by itself does not afford us any valuable result:

It must be distinguished between an individual (single) and an ensemble measurement (i.e. in principle, an infinitely large number of repeated measurements on identically prepared objects). Performing a single measurement on the object, a totally unpredictable value is observed in general. (Vogel et al. 2001, 225)

In this way we need an experimental procedure affordig us to obtain the relative rate at which a particular value for a quadrature component q_θ is observed, i.e. the probability distribution $\text{pr}(q, \theta)$, where θ is the relative phase between signal and local oscillator. Thus, making a large number of measurements of the observable q_θ yields $\text{pr}(q, \theta)$, i.e. the probability distribution of its eigenvalues. In general the experimental procedure goes as follows:

The phase θ can be easily varied using a piezo-electric translator. To measure quadrature distributions, we may fix the phase angle θ and perform a series of homodyne measurements at this particular phase to build up a quadrature histogram $pr(q, \theta)$. Then the [local oscillator] phase should be changed in order to repeat the procedure at a new phase, and so on [, in a way that we obtain results for a set of different phase angles between 0 and π]. (Leonhardt 1997, 99)

The probability distribution $pr(q, \theta)$ is equal to $\langle q|U(\theta)\rho U^*(\theta)|q\rangle$, where ρ is the density operator (which provides the most general description of a quantum state), and $U(\theta) = \exp(-i\theta n)$ is the phase-shifting operator (where n is the photon number operator). From the experimentally obtained probability distribution $pr(q, \theta)$, it is possible to reconstruct the so-called Wigner function $W(q, p)$, which is closely related to the density operator. In reality both can be seen as “one-to-one representations of the quantum state” (Leonhardt 1997, 40). The interesting part comes now. In the case of the vacuum field (like in all others), we have a good agreement between the experimentally reconstructed Wigner function and the theoretical Wigner function (Leonhardt 1997, 46-47). In particular, if we consider the quadrature wave function of the vacuum state

$$\psi_0(q) = \pi^{-1/4} \exp\left(-\frac{q^2}{2}\right) \quad (3.31)$$

the measured quadrature probability distribution $|\psi_0(q)|^2$ “is approximately Gaussian and already follows the theoretical expectation” (Leonhardt 1997, 23).

In general there is a good agreement between the theoretical predictions for several quantum states of light (e.g. single-photon Fock states and squeezed states) and the experimental results (e.g. Breitenbach et al. 1997; Bertet et al. 2002). This gives assurance that the results obtained in the case of the vacuum state can be taken to be a property of the vacuum state and not as resulting from some other physical origin, for example, from the material of the photodetectors. I mention this because in these experiments we are considering a high intensity field that is treated, by correspondence arguments, as a classical field that interferes with the vacuum field, producing two different beams. As I said, each beam is directed to a photodetector. Ideally each of the two photodetectors will produce a photocurrent that is proportional to the number of photons of the beams striking each one. It would seem that we are detecting individuated photons due to the vacuum field, which would imply that the photodetectors are receiving momentum and energy from the vacuum. However we must recall that what is being detected are the beams resulting from the interference of (what can be considered) a classical field and the vacuum field. The possible ‘photons’ from the vacuum are not ‘differentiated’ from the ‘photons’ from the classical field. We must take into account the usual identification of a classical field with a quantum coherent field with a large expectation value of the photon number operator. The coherent state does not have a definite number of photons. In fact it can be defined by an infinite expansion in terms of photon number states, that is, by taking into account photon states corresponding to an infinite number of photons. Due care is needed in the physical interpretation of this situation, in particular in what concerns the possibility of detection of photons in the ground state, which I consider not to be possible (and theoretically is nonsense), and this makes the usual quantum theory of the photodetectors based on the idea of photon absorption problematic (Vogel et al. 2001, 169-190).

More than questioning the experimental results regarding the vacuum electromagnetic field, this indicates a need to revise the theoretical treatment of the interaction of photodetectors with the quantized electromagnetic free field. However, I believe that there is no final and conclusive approach to address this problem. In quantum electrodynamics we face an ‘intrinsic’ limitation in the description of the interaction of radiation and matter (since we are only able to make approximate calculations) that lead me to consider that we cannot go beyond a ‘model’ level of description of the interaction between the quantized electromagnetic field and the photodetectors. However, this problem is well beyond the scope of the present paper.

As I said, what gives me confidence on the interpretation of the experimental results for the quantum electromagnetic vacuum is the coherence between the interpretation of $\langle 0 | \mathbf{E}^2 | 0 \rangle$ and the experimental results obtained for different quantum states of light, including the vacuum state.

4 Further remarks

In this work I am more interested in a tentative clarification (or at least in a contribution towards it) of the concept of vacuum in quantum electrodynamics than in possible philosophical ramifications of the view presented. The lengthy, but necessary, technical discussions presented show that there are intricate aspects that make it very difficult to have any strong metaphysical commitment regarding the concept of vacuum (e.g. of an ontological nature); more than explore possible metaphysical consequences of the conceptual analysis being presented, the intention here is to provide a ‘frame’ within which to make clear what we should not attribute to the concept of vacuum and what we might attribute to it even if with some reservations.

We have seen that there are subtle theoretical and experimental aspects related to the ground state of the quantized electromagnetic field, which represent a clear departure from the ‘nothingness’ of the classical concept of vacuum. However, we must be careful not to ascribe too much to the ground state of the quantized field. One difference with the classical theory is that when considering a charged particle we must consider it to be at least in ‘interaction’ with the ground state of the quantized electromagnetic field. In a certain sense the quantized radiation and matter need a more integrated description. As Milonni stressed, “without [the source-free field] the whole quantum theory of a charged particle in vacuum becomes inconsistent” (Milonni 1994, 125).

As we have seen the other aspect in which the quantum concept of vacuum has a clear departure from the classical ‘nothingness’ is at the experimental level. We see experimental results that for their interpretation it is necessary to take into account the concept of quantum vacuum. The view defended here is that we can deflate the so-called experimental vacuum effects to a simple aspect common to any n-photon state of the field: a non-vanishing variance of the electric and magnetic fields.⁵ That is, there are

⁵ We must recall that this variance is related with measurements made on identically prepared systems not one individual system. As mentioned, there is a tendency in the literature to refer to the non-vanishing variance as ‘fluctuations’ of the vacuum state (Sakurai 1967, 32-33; Aitchison 1985, 346-247). This view is misleading since the non-zero variance of the ground state of the quantized electromagnetic field cannot be related to a fluctuation in time: “there is no time evolution of this vacuum state” (Rugh and Zinkernagel 2002, 673). Considering measurements made on equally prepared systems, they will show fluctuations in the results of the successive observations – according to the interpretation of the theory

no dynamical effects of the vacuum state. In this way the formal considerations (pointing to the need of an external ‘independent’ quantized electromagnetic field even if in its ground state) and the experimental results (giving observational meaning to the mathematical expression of the variance even in the case of the vacuum state) are consistent with each other and point to a concept of vacuum that has theoretical and experimental relevance. However the only experimental results we can attribute with relative security to the vacuum state relate to a non-vanishing variance and not some spectacular dynamical effects. This implies that we cannot attribute any feature to the vacuum state that makes it ‘special’ when compared with other states of the quantized electromagnetic field. In this way whatever metaphysical ramifications might there be related to the concept of vacuum they do not go beyond the ones we might endorse in relation to any other state of the quantized electromagnetic field, which is not an unexpected result since the quantized electromagnetic field is a more fundamental concept than the particular state it might be in. Looking from this perspective, if it was the case that the vacuum state appeared simply as an artifact of the theory (not being possible to give an observational meaning to its variance) this would create an awkward situation regarding the concept of quantum field. We would be in a situation in which the physical-mathematical structure of the theory would be inconsistent with the experimental results. On one side, for example a one photon state of the quantized electromagnetic field would have experimental significance (and we could relate the variance to observed outcomes of measurements made on the electromagnetic field in this state) while on the other side, another state of the field, the ground state, would be an artifact (and in this case the variance would be a physically meaningless mathematical expression). That is not the case.

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followed here (Isham 1995, 80-81; Peres 1995, 24-26; Ballentine 1998, 225-227, Falkenburg 2007, 205-207). The non-vanishing variance is not a temporal property of one single system. We observe a statistical fluctuation on the results of measurements made on equally prepared systems, not a temporal fluctuation of the same system.

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